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**253. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.**

Find the maximum number of real points of inflection in each of the quartic curves $y^2=\alpha x^4\pm x^2+\beta$, and find the necessary and sufficient relations between $\alpha$ and $\beta$ for the existence of this number of points of inflection.

**Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.**

$y=\sqrt{[\alpha x^4\pm x^2+\beta]}$. Then $\dfrac{dy}{dx}=\dfrac{2\alpha x^3\pm x}{\sqrt{[\alpha x^4\pm x^2+\beta]}}$.

$$\frac{d^2y}{dx^2}=\frac{2\alpha^2x^6\pm3\alpha x^4(2\alpha-1)+6\alpha^2\beta x^2\pm\beta}{(\alpha x^4\pm x^2+\beta)^{\frac{3}{2}}}=0.$$

$$\therefore x^6\pm\frac{3(2\alpha-1)}{2\alpha}x^4+3\beta x^2\pm\frac{\beta}{2\alpha^2}=0.$$

Let $x^2=z$, $\dfrac{3(2\alpha-1)}{2\alpha}=a$, $3\beta=b$, and $\dfrac{\beta}{2\alpha^2}=c$. $\quad\therefore z^3\pm az^2+bz\pm c=0$.

Let $z=u\mp\frac{1}{3}a$. Then $u^3-(\frac{1}{3}a^2-b)u\mp(\dfrac{ab}{3}-\frac{2}{27}a^3-c)=0$, or $u^3-Au\mp B=0$.

$\therefore$ When $A^3/B^2<27/4$, there are three real unequal roots.

$$\therefore\ \frac{[(2\alpha-1)^2-4\alpha^2\beta]^3}{[6\alpha^2\beta(2\alpha-1)-2\alpha\beta-(2\alpha-1)^3]^2}<1.$$

Then $y^2=\alpha x^4+x^2+\beta$ can have six real points of inflection, while $y^2=\alpha x^4-x^2+\beta$ can have but four. The values of $\alpha$ and $\beta$ can reduce this number in both cases.

## NUMBER THEORY AND DIOPHANTINE ANALYSIS.

**122. Proposed by DR. L. E. DICKSON, The University of Chicago.**

If $p$ is prime, $(p^4-1)(p^2-1)$ has no factor of the form $1+p^3x$, $x>0$, if $p>2$; $(p^6-1)(p^4-1)(p^2-1)$ has no factor of the form $1+p^5x$, $x>0$.

**Solution by H. S. VANDIVER, Bala, Pa.**

I. Assume that $(p^4-1)(p^2-1)$ is divisible by $1+p^3x$ where $x$ is not zero, $p$ *any* integer, not zero. By inspection we see that there exists a positive integer $y$, not zero, such that

$$(p^4-1)(p^2-1)=(1+p^3x)(1+p^2y).$$

Now write the left hand member in the scale of notation whose radix is $p$. Then

$$p^5(p-1)+p^4(p-2)+p^3(p-1)+p^2(p-1)+1$$
$$=1+p^3x+p^2y+p^5xy.$$

From which it is evident that $xy \ngtr p-1$, and $x$ and $y$ are each less than $p$. The right hand member is therefore also expressed in the $p$-scale of notation, and the two expressions are identical. Whence, equating coefficients of $p^4$, we obtain $p=2$, and $(2^4-1)(2^2-1)=(1+2^3)(1+2^2)$. Otherwise the assumed equality is impossible. Note that $p$ is not necessarily prime.

II. Assume that $(p^6-1)(p^4-1)(p^2-1)$ has a factor of the form $1+p^5x$, $x>0$, then there is necessarily an integer $y>0$, such that

$$P=(p^6-1)(p^4-1)(p^2-1)=(1+p^5x)(-1+p^2y)\ldots(1).$$

Whence, by expansion and division by $p^2$,

$$y=1+p^2+p^3x+p^{10}-p^8-p^6-p^5x\ldots(2),$$

and $y$ is therefore of the form $1+p^2+p^3M$.

Since $y>0$, then $M \geqq 0$. If $M=0$, $y=p^2+1$, and from (2),

$$x=p^3-\frac{2p^5}{p^4+p^2-1}.$$

Now $p^5$ is prime to $p^4+p^3-1$, and this is contrary to the hypothesis that $x$ is an integer. Assume, then, $M>0$. In this case, evidently $y>p^3$. As a consequence, $x<p^2$, for if $x \geqq p^2$ then from (1), $P>(1+p^2)(-1+p^5)$, and by expanding and dividing by $p^2$, the result may be written $p^8-1+p^5(p-1)+p^2(p-1)<0$, which is absurd. In a similar way, it may be shown from (1), that $y<p^5$, for all values of $x$, not zero.

From (2), $y-(1+p^2+p^3x)$ is divisible by $p^5$. Now since $x<p^2$, $1+p^2+p^3x<p^5$, and since $y<p^5$, we must have $y=p^3x+p^2+1$. Comparing with (2), we obtain

$$p^5-p^3-p=x(p^3x+p^2+1),$$

and evidently $x \equiv 0 \pmod{p}$. Hence the right hand member is greater than $p^5$. But the left hand member is less, so (1) is impossible. Note again that $p$ is not necessarily prime.